Length Element :-

$$\vec{r} = \vec{r}(u_1, u_2, u_3)$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial u_1} du_1 + \frac{\partial \vec{r}}{\partial u_2} du_2 + \frac{\partial \vec{r}}{\partial u_3} du_3$$

$$d\vec{r} = \hat{e}_1 h_1 du_1 + \hat{e}_2 h_2 du_2 + \hat{e}_3 h_3 du_3$$

$$\because \quad \frac{\partial \vec{r}}{\partial u_1} = \hat{e}_1 h_1, \quad \frac{\partial \vec{r}}{\partial u_2} = \hat{e}_2 h_2, \quad \frac{\partial \vec{r}}{\partial u_3} = \hat{e}_3 h_3$$

$$\boxed{d\vec{r} = h_1 du_1 \hat{e}_1 + h_2 du_2 \hat{e}_2 + h_3 du_3 \hat{e}_3}$$

Area element :-

$$d\vec{A}_1 = h_2 du_2 \hat{e}_2 \times h_3 du_3 \hat{e}_3$$

$$d\vec{A}_1 = h_2 h_3 du_2 du_3 (\hat{e}_2 \times \hat{e}_3)$$

$$\boxed{d\vec{A}_1 = h_2 h_3 du_2 du_3 \hat{e}_1}$$

Similarly.

$$d\vec{A}_2 = h_3 h_1 du_3 du_1 \hat{e}_2$$

$$d\vec{A}_3 = h_1 h_2 du_1 du_2 \hat{e}_3$$

Volume Element :-

$$dV = h_1 dU_1 \hat{e}_1 \cdot [h_2 dU_2 \hat{e}_2 \times h_3 dU_3 \hat{e}_3]$$

$$dV = h_1 dU_1 \hat{e}_1 \cdot h_2 h_3 dU_2 dU_3 \hat{e}_1$$

$$\boxed{dV = h_1 h_2 h_3 dU_1 dU_2 dU_3} \quad (\because \hat{e}_1 \cdot \hat{e}_1 =$$

In Cartesian Co-ordinate :- (x, y, z)

Length Element.

$$d\vec{r} = h_1 dU_1 \hat{e}_1 + h_2 dU_2 \hat{e}_2 + h_3 dU_3 \hat{e}_3$$

$h_1 = 1, \quad h_2 = 1, \quad h_3 = 1$

$dU_1 = dx, \quad dU_2 = dy, \quad dU_3 = dz$

$\hat{e}_1 = \hat{a}_x \quad \hat{e}_2 = \hat{a}_y, \quad \hat{e}_3 = \hat{a}_z$

$$\boxed{d\vec{r} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z}$$

Area element

$$d\vec{A}_1 = dz\,dy\,\hat{a}_x$$

$$d\vec{A}_2 = dz\,dx\,\hat{a}_y$$

$$d\vec{A}_3 = dx\,dy\,\hat{a}_z$$

volume element

$$dV = h_1 h_2 h_3 \, du_1 \, du_2 \, du_3$$

$$\boxed{dV = dx\,dy\,dz}$$

Cylindrical co-ordinate :- (ρ, ϕ, z)

length element :

$$d\vec{r} = h_1 du_1 \hat{e}_1 + h_2 du_2 \hat{e}_2 + h_3 du_3 \hat{e}_3$$

$$h_1 = 1, \quad h_2 = \rho, \quad h_3 = 1$$

$$d\vec{r} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

area element :

$$d\vec{A}_1 = h_2 h_3 \, du_2 \, du_3 \, \hat{e}_1$$

$$d\vec{A}_1 = \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$d\vec{A}_2 = d\rho \, dz \, \hat{a}_\phi$$

$$d\vec{A}_3 = \rho \, d\rho \, d\phi \, \hat{a}_z$$

volume element

$$dV = h_1 h_2 h_3 \, du_1 \, du_2 \, du_3$$

$$dV = \rho \, d\rho \, d\phi \, dz$$

Spherical co-ordinate :⇒ (r, θ, ϕ)

$h_1 = 1, \quad h_2 = r, \quad h_3 = r\sin\theta$

Length element:

$$d\vec{r} = h_1 du_1 \hat{e}_1 + h_2 du_2 \hat{e}_2 + h_3 du_3 \hat{e}_3$$

$$d\vec{r} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

Area element:

$$d\vec{A_1} = h_2 h_3\, du_2\, du_3\, \hat{e}_1$$

$$d\vec{A_1} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$$

$$d\vec{A_2} = r\sin\theta\, d\phi\, dr\, \hat{a}_\theta$$

$$d\vec{A_3} = r\, dr\, d\theta\, \hat{a}_\phi$$

volume element:

$$dV = h_1 h_2 h_3\, du_1\, du_2\, du_3$$

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$$